Cryogenic Instrumentation

Module 6: Cryogenic Engineering ME 467

Temperature Measurement

Is measured indirectly (unlike say, length, mass etc.) from the change of other properties:

- Length of a Hg column
- The electrical resistance of a Platinum wire
- Pressure of a near ideal gas
- Thermo electric emf
- Equilibrium pressure of a gas above a boiling liquid
- The difference in thermal expansion of two metals in a composite beam
- The speed of sound in a gas
- The magnetic susceptibility of paramagnetic material etc.

Pay off functions:

- Accuracy – the departure from thermodynamic temperature scale
- Sensitivity – the rate of change of indicating property with temperature
- Reproducibility – the range of indications of temperature when several measurements are made of the same temperature.
- Stability – changes in the indication of the thermometer over a period of use

Temperature scales

- **Thermodynamic Temperature Scale,** $\dfrac{T_{cold}}{T_{hot}} = \dfrac{Q_{rejected\ @Tc}}{Q_{absorbed\ @Th}}$ (*refer, 2nd Law of TD/ Carnot Cycle*)

- *Fixed point:* triple point of water (273.16 K) (*refer, zeroth Law of TD/ Thermometry*)

- At 7th General Conference on Weights and Measures (1927), an **International Temperature Scale (ITS-27)** was adopted.
    - A practical temperature scale that closely approximates the thermodynamic temperature scale and was easily reproducible.
    - *Fixed points:* 6 fixed points with assigned numerical values
    - Defined down to only the NBP of O_2

- In the 11th conference in 1960, the triple point of water was selected as the standard fixed point, **International Practical Temperature Scale (IPTS-48)**

- The 13th conference extended the range to the triple point of H_2 (13.81 K) – **ITPS-68**

The standard measurement instruments,

- **Strain free Platinum resistance thermometer**
  - between triple point of H_2 (13.84 K) and freezing point of Antimony (903.89 K)
- **Germanium resistance thermometer**
  - From 4.24 K to 13.84 K
- **He^4 vapor pressure scale**
  - From 1.5 K to 4.24 K
- **He^3 vapor pressure scale**
  - From 0.8 K to 1.5 K
- **Magnetic scale**
  - From 0.006 K to 0.8 K
  - Based on magnetic susceptibility of Cerium Magnesium Nitrate.
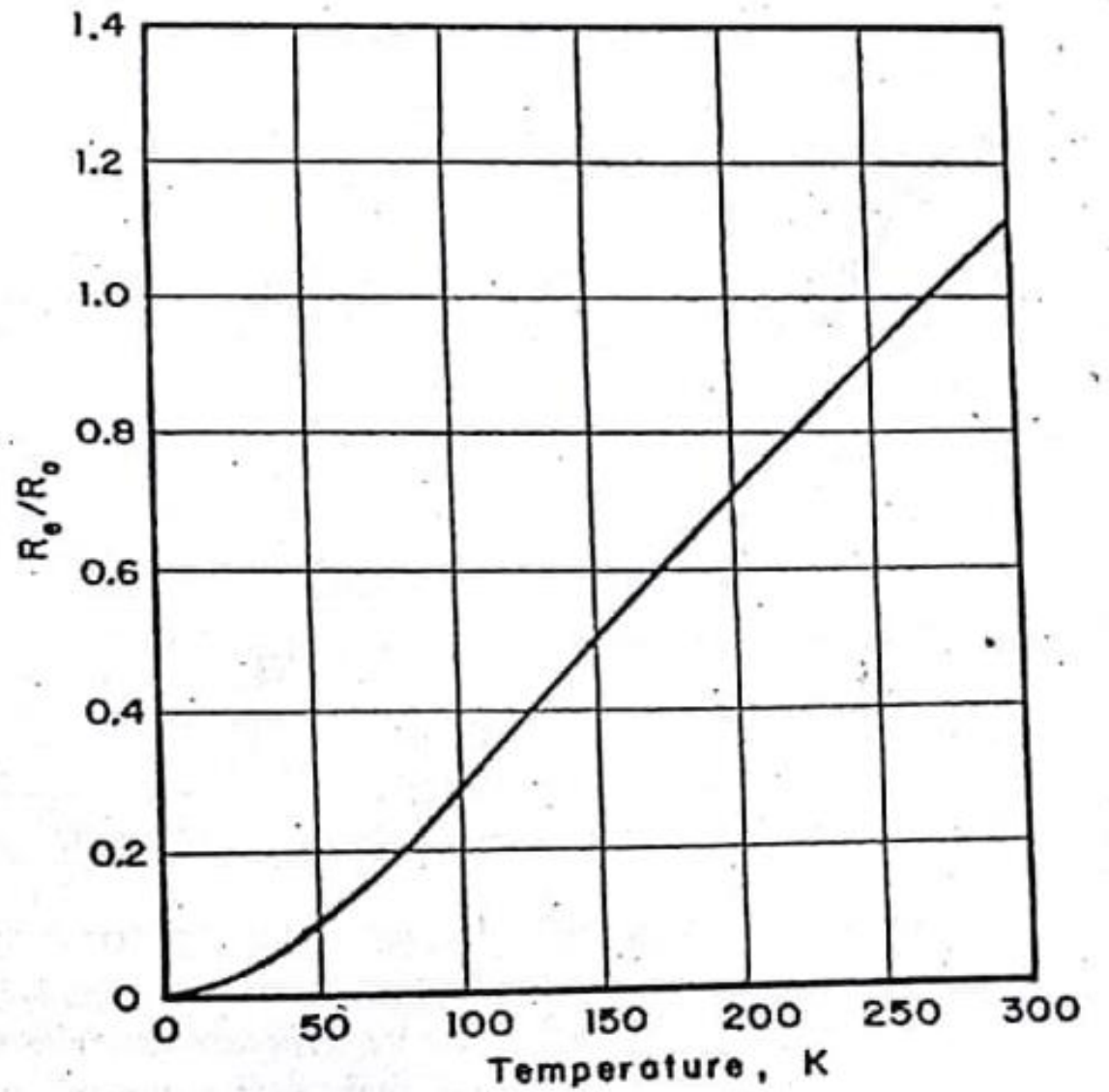
Metallic resistance thermometers

- Temperature variation of electrical resistivity of conductors and semi conductors is the basis

- In addition to Platinum, copper, lead and indium have been used.

- A linear variation of resistance is desirable.

- Typically calibrated by Callendar-van Dusen equation

$$R_e/R_0 = 1 + At + Bt^2 + Ct^3(t - 100)$$

- Where t is in ^{0}C; A, B, C are constants found by calibration at 3 standard temperatures, R_0 is the resistance at 0 ^{0}C and R_e is the resistance at t ^{0}C .

- It is important the resistance wire be strain free
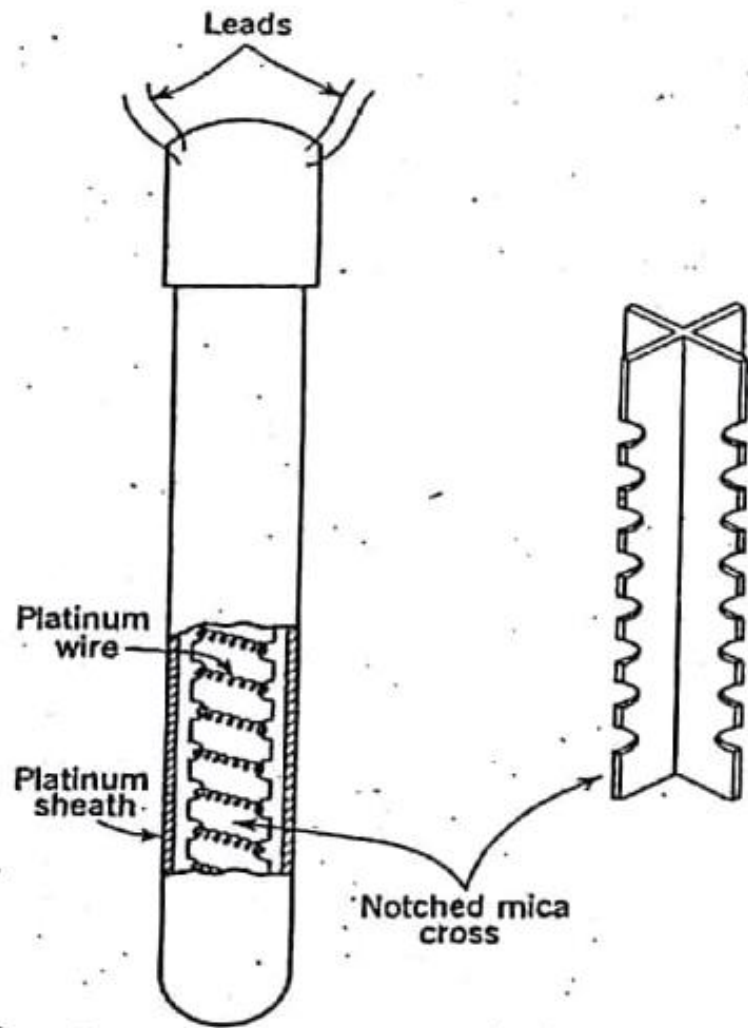
Platinum resistance thermometer

A platinum resistance thermometer yields a resistance reading of 38.6 ohms a certain temperature. If the electric resistance at 0 ^{0}C is 100 ohms and the constants A, B, C are respectively 3.946 x10^{-3} ^{0}C^{-1}, -1.108 x10^{-6} ^{0}C^{-2}, and 3.33 x10^{-12} ^{0}C^{-4} ,Determine corresponding temperature indication.

- Hint: Solve iteratively; start by solving for t, neglecting B & C terms, in the first iteration. In the second iteration, put the value of t previously obtained in the B & C terms to solve for t again. Repeat until the value t in successive iterations are very close (~ 0.01 ^{0}C)

- Ans: t = -150 ^{0}C

- Sensitivity, S_0 = 0.4211 Ω/^{0}C

$$R_c/R_0 = 1 + At + Bt^2 + Ct^3(t - 100) \qquad S_0 = \frac{dR_c}{dT} = R_0[A + 2Bt + Ct^2(4t - 300)]$$

Semi Conductor Resistance Thermometers

- Electrical resistivity is inversely proportional to temperature at high temperatures
- At low temperatures, electron furnishing (donors) or accepting (acceptors) impurities aid conduction.
- Germanium semiconductors are the most widely used.
  - A single crystal of Ge is doped with arsenic, gallium or antimony to obtain desired resistance characteristics.
  - Ge is sealed inside a metal case and four leads are attached (two for supplying operating current, two for measuring voltage)
  - Typical operating range between 1.5 K to 100 K
- Carbon resistance thermometers
  - High sensitivity, low cost, small size and relatively simple temperature-resistance curve

Thermocouples

- Can be used as in regular temperature ranges.
- But, during calibration, reference temperature should be selected close to the measurement range (LN_2, LH_2, LHe temperatures, depending on the range of temperature measurement, but not the ice point)
  - High temperature difference produce high emf which will require precision reading of emf for precise measurement of temperature.
  - Heat conduction down the thermocouple wire is also reduced.
- If emf is very low thermocouples connected in series (**Thermopile**) can be used.
- Conduction down to the cold end bead can warm up the bead and show erroneous reading of temperature. This can be avoided by insulation
- Calibration equation is given by,

$$e = a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

Constant Volume Gas Thermometer

- A bulb containing the gas is inserted in the region whose temperature is to be measured. And the pressure is measured.

- Calibrated by measuring gas pressure at a standard temperature.

- Assuming ideal gas relation,

$$T = p(T_s/p_s)$$
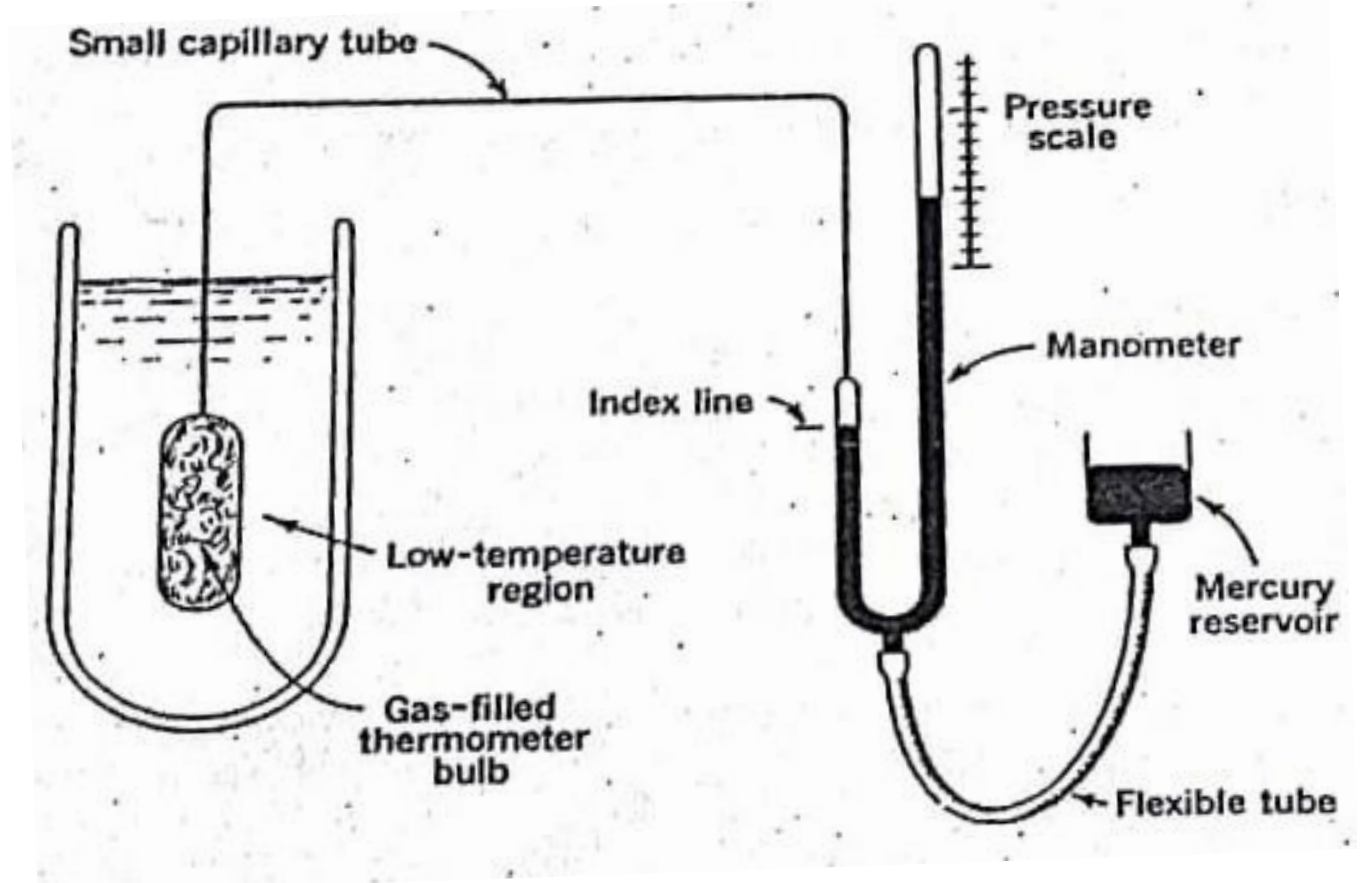
- We get a linear dependence of temperature on pressure.
- Considering the dead volume gas,

$$m = \rho V + \rho_0 V_0 = \rho_s V + \rho_{0s} V_0$$

where m = mass of gas within the thermometer
ρ = density of gas within the thermometer bulb at temperature T
ρ_0 = density of gas in dead volume at temperature T_0
ρ_s = density of gas at the standard temperature T_s
V = volume of the thermometer element
V_0 = dead volume

$$pV/RT + pV_0/RT_0 = p_s V/RT_s + p_s V_0/RT_0$$

- Solving for temperature,

$$T = \frac{p(T_s/p_s)}{1 + (1 - p/p_s)(V_0/V)(T_s/T_0)} = K_1 p(T_s/p_s)$$

- Where K is the correction factor

- In addition to dead volume correction, real gas effect may be considered,
- Virial expansion (real gas equation) is given by,

$$pv = RT[1 + B(T)p + C(T)p^2 + D(T)p^3 + \ldots]$$

- Where B, C, D etc. are virial coefficients
- Neglecting from the third term onwards,

$$\rho = p/RT[1 + B(T)p]$$

- Substituting in the mass equation and solving for temperature we get,

$$T = K_1(1 + K_2)(T_s/p_s)p$$

Vapor Pressure Thermometer

- Vapor Pressure (saturation pressure) is a function of temperature.

- Bulb contains a pure gas that can be condensed in the temperature range of interest.

- High sensitivity.

- No dead volume correction or real gas correction is required.
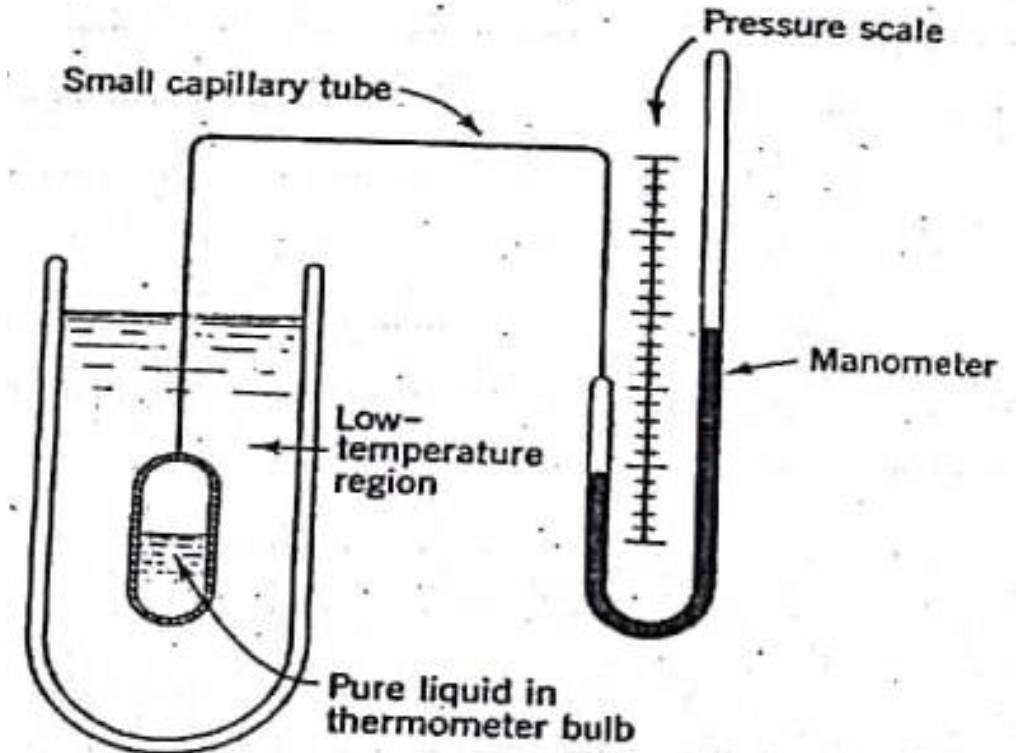
- Disadvantage: Limited range (can be made to use from the triple point to the critical point of the given pure substance)

- When H2 is used as thermometric fluid, catalyst must be used to promote conversion from normal to equilibrium hydrogen.

- Calibration equation is given by,

$$\ln(p/p_0) = C_1 - C_2/T - C_3 \ln(T/T_0) - C_4 T + C_5 T^2$$

- Where P_0 is atm pressure and T_0 the normal boiling point. C is a constant.

Example 6.5. Determine the temperature indication for a liquid-hydrogen vapor-pressure thermometer when the pressure reading is 14 kPa (2.03 psia).

$$\ln(p/p_0) = C_1 - C_2/T - C_3 \ln(T/T_0) - C_4 T + C_5 T^2$$

- Hint: Newton's method: iterative

$$T_2 = T_1 - f(T_1)/f'(T_1)$$

- Ans: T = 15.09 K

Table 6.4. Constants in the vapor-pressure relationship, $\ln(p/p_0) = C_1 - C_2/T - C_3 \ln(T/T_0) - C_4 T + C_5 T^2$ where $p_0 = 101.325$ kPa, $T_0 = 77.344$ K for N_2 and 90.188 K for O_2, and the temperature T is in kelvins. Log base e is used.

	Hydrogen	Neon	Nitrogen	Oxygen
C_1	3.940796	10.618417	13.569758	13.726967
C_2	101.33783	244.96075	930.15333	1076.35667
C_3	0	0	2.3668	1.664512
C_4	−0.0543201	0.0848111	0.0328844	0.0304241
C_5	-1.10563×10^{-4}	9.78350×10^{-4}	1.67138×10^{-4}	1.16981×10^{-4}

Magnetic Thermometer

- For temperature measurements below 1.0 K
- Magnetic materials obey Curie's law (at relatively high temperatures and weak fields)

$$x = J/H = C/T$$

- X – magnetic susceptibility, J – magnetic moment per unit mass, H – applied magnetic field intensity, C – Curie's constant (material property)
- Not applicable at very low temperatures.

Calibration

- 1 – 2, isothermal magnetisation (> 1 K, measured by a He – vapour pressure thermometer). The entropy can be determined by the Brillouin expression from T, J, H etc.

- 2 – 3, reversible adiabatic removal of magnetic field. S3 = S2

- We define a magnetic temperature $T^* = \mathcal{C}/\chi$

- T* not equal to T (not Curie's Law)

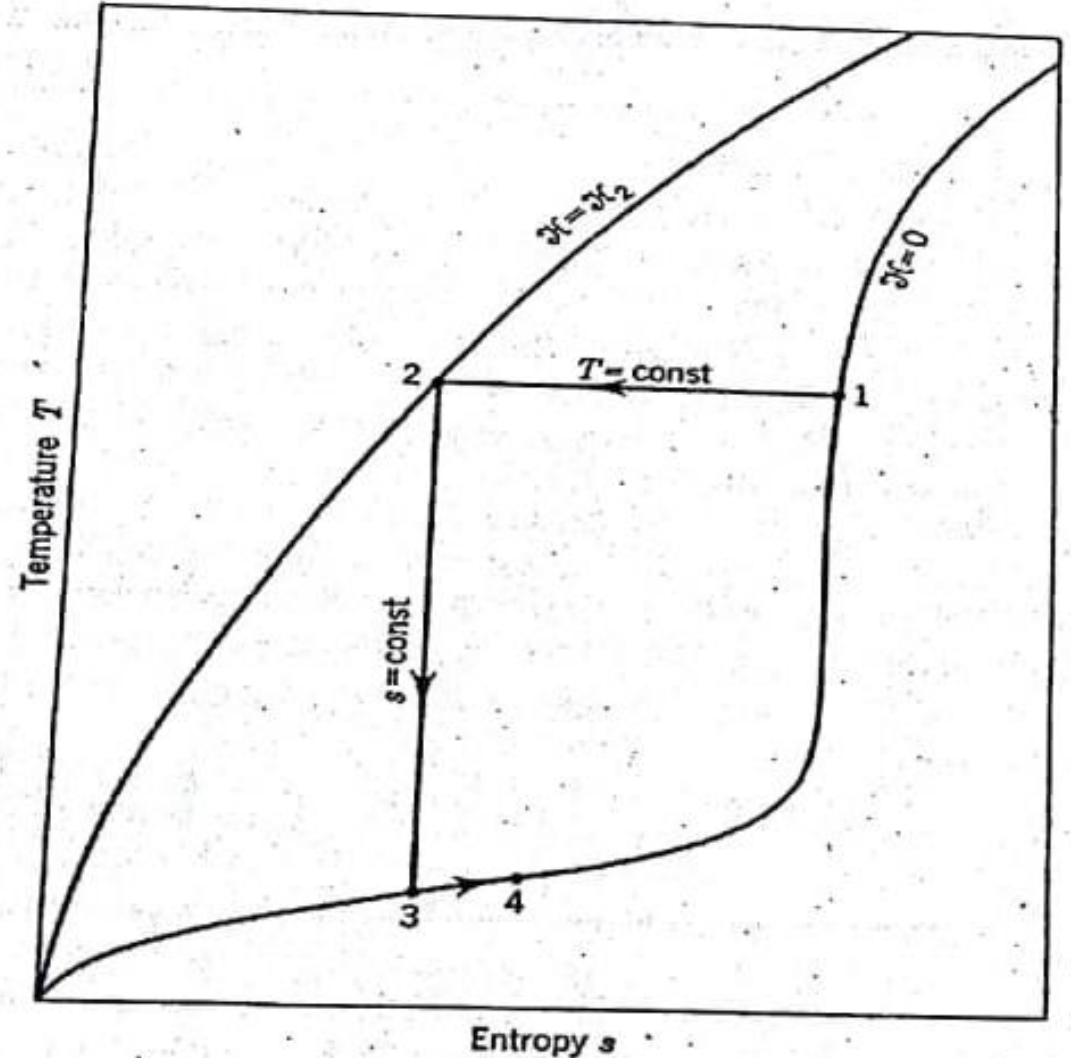- S vs T* plot formed after a series of such demagnetisation

Fig. 6.7. Calibration process for a magnetic thermometer.

- 3 – 4 : addition of energy at zero magnetic field
- We can measure the change in T* due to change in the internal energy.

$$T\,ds = du - \mu_0 \mathcal{H}\,dJ$$

$$T = \left(\frac{\partial u}{\partial s}\right)_{\mathcal{H}=0} = \frac{(\partial u/\partial T^*)_{\mathcal{H}=0}}{(\partial s/\partial T^*)_{\mathcal{H}=0}}$$

- T vs T* calibration curve can be constructed (some examples below)

1. Cerium magnesium nitrate (Hudson and Pfieffer 1972), for $T \geq$ 0.004 K:

$$T^* = T + (0.236 + 0.004137/T)(10^{-3}) \qquad (6.36)$$

2. Chromic methylammonium alum (Gardner and Kurti 1954), for $T \geq$ 0.070 K:

$$T^* = T + 0.00250 + 0.002422/T \qquad (6.37)$$

3. Chromium potassium alum (Ambler and Hudson 1957), for $T \geq$ 0.10 K:

$$T^* = T + 0.000862 + 0.002057/T \qquad (6.38)$$

Flow-Rate Measurement

- Orifice meter
- Venturi meter
- Turbine flow meter

Orifice Meter

- Calibration curve for water can be applied directly LO2, LN2, LH2 (single phase)
- Upstream straight line length has to be at least 10 times diameter of pipe & downstream length 5 times.
- Discharge coefficient accounts for losses. Is determined by calibration or ISO-ASME orifice coefficient expression.
- Pressure taps are D and 0.5 D up and downstream respectively.
- Low cost and size, but large pressure drops
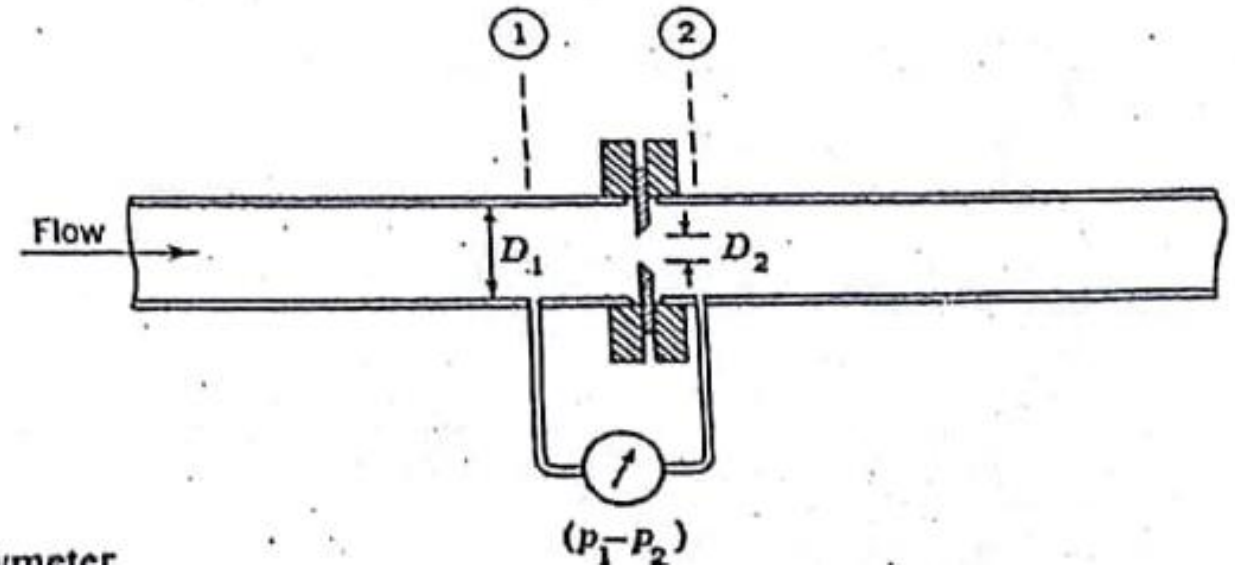
$$\dot{m} = C_d C_a A_0 (2g_c \rho \Delta p)^{1/2} \qquad C_a = (1 - \beta^4)^{-1/2}$$

C_d = discharge coefficient
C_a = velocity-of-approach coefficient
A_0 = area of the orifice
ρ = fluid density
Δp = pressure drop across the orifice

re $\beta = D_0/D_p$ = orifice diameter ratio
D_0 = orifice diameter
D_p = pipe inside diameter
$N_{Re} = D_p G/\mu$ = pipe Reynolds number
$G = \dot{m}/A_p$

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.1840\beta^8 + 0.0390\beta^4/(1 - \beta^4)$$
$$- 0.01584\beta^3 + 91.71\beta^{2.5} N_{Re}^{-0.75}$$

Venturi Meter

- Avoids large pressure drops of an orifice.
- Sharp ($20^0 - 22^0$) convergent section, straight throat(~0.5D), gradual ($5^0 - 7^0$) diverging section.
- Pressure taps @ 0.5D before venturi and @ the centre of the throat.
- Flow rate measured by the same expression as the orifice meter.
- Cd is almost unity. Determined by calibration or by the given expression.
- Cavitation chances are high.
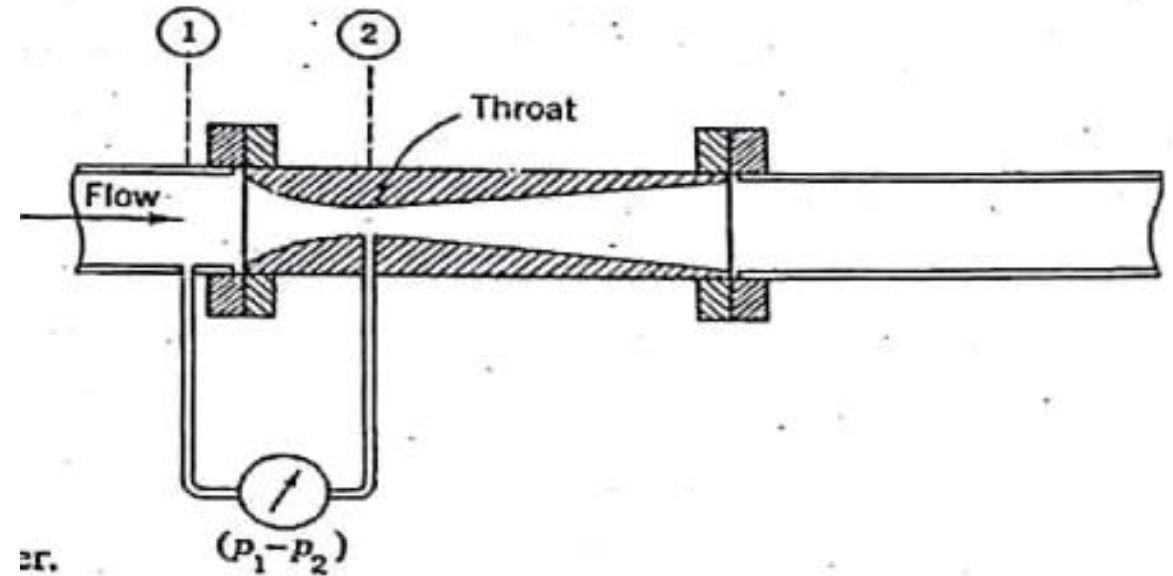
1. For $3000 \leq N_{Re} \leq 2 \times 10^5$,

$$C_d = \frac{\log_{10} N_{Re}}{0.60 + 0.90 \log_{10} N_{Re}}$$

2. For $N_{Re} > 2 \times 10^5$,

$$C_d = 0.988$$

Turbine Flowmeter

- Free spinning turbine in the centre of the flow.

- A magnetic pick-up sensor generates pulses as the blades cross its field, frequency of which is indicated by an oscilloscope or an EPUT meter.

- At Re>6000, volume flow rate is related to speed n of the turbine by the given equation.

- Viscosity correction of calibration factor required for low speeds

- Density (and hence 2 phase flow) doesn't affect the performance

- Low inertia of turbine allows good dynamic response (for transient flows)
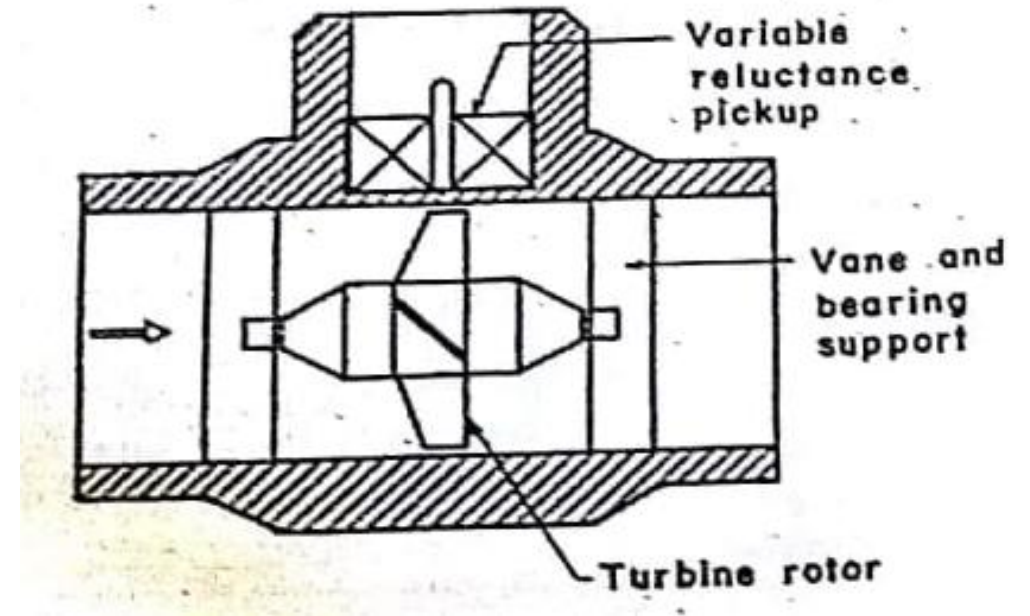
$$\dot{V} = \pi D_b A_{ff} n / \tan \theta_b = Kn$$

D_b = rotor blade-tip diameter
θ_b = angle between the blade and the meter centerline
A_{ff} = free-flow area through the turbine
$A_{ff} = \frac{1}{4}\pi(D^2 - D_h^2)[1 - 2mt/\pi(D + D_h) \cos \theta_b]$
D_h = rotor hub diameter; D = meter bore diameter
m = number of blades
t = blade thickness

Fluid Quality Measurement

- Important to measure quality for flow measurement (density can be found only by knowing quality)

- The dielectric constant of the fluid varies with quality.

- A concentric cylinder capacitor is formed by keeping a cylindrical rod inside the pipe suspended through insulators (with the pipe wall and the rod acting as parallel plates)

- The annulus area must be same as the pipe area to avoid acceleration or deceleration of fluid.

- The probe capacitance can be related to quality by static calibration or the following method,
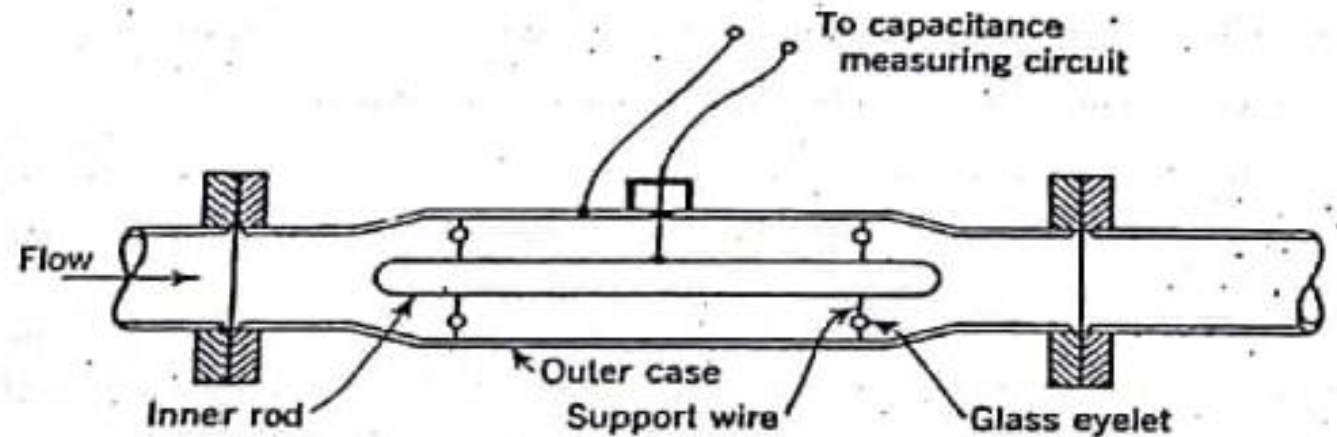
- The capacitance (C) of a concentric cylinder capacitor long enough that end effects can be neglected is known.
- The dielectric constant (ϵ) is related to the volume fraction (y) of liquid and vapor.
- The density (ρ) is also related to volume fraction.
- Thus from volume fraction and density quality (x) can be determined.
- ρ & ϵ also depends on Pressure. Thus the calibration must be carried out for the operating pressure.
- If is ϵ not known Clausius-Mossotti equation can be used. (M – molecular weight, α - molar polarisation)

$$C = 2\pi L \epsilon \epsilon_0 / \ln(D_o/D_i)$$

L = length of the cylinder
ϵ = relative dielectric constant of the material in the annulus
ϵ_0 = 8.8542 × 10^{-12} F/m = permittivity of free space

$$\epsilon = y\epsilon_f + (1 - y)\epsilon_g$$

ϵ_f = dielectric constant of saturated liquid
ϵ_g = dielectric constant of saturated vapor
$y = V_f/(V_f + V_g)$ = liquid volume ratio

$$\rho = y\rho_f + (1 - y)\rho_g$$

$$x = m_g/m = V_g\rho_g/(V_f + V_g)\rho = (1 - y)(\rho_g/\rho)$$

Clausius-Mossotti equation,

$$\frac{M(\epsilon - 1)}{\rho(\epsilon + 2)} = \alpha_M$$

Liquid Level Measurement

- Hydrostatic Gauges

- Electric Resistance Gauges

- Capacitance Liquid-level Probes

- Thermodynamic Liquid-Level Gauges

Hydrostatic Gauges

Vacuum space

Vapor L_g

Liquid L_f

Δp Pressure gauge

Capillary tube

- Differential pressure measuring instruments like Bourdon gauge, U-tube Manometer, diaphragm transducer etc. Can be used.

- Pressure pickups (liquid) are constructed with goosenecks to avoid liquid drain.

- Problems:
  - Low sensitivity (LH$_2$, LHe)
  - Pressure oscillations.
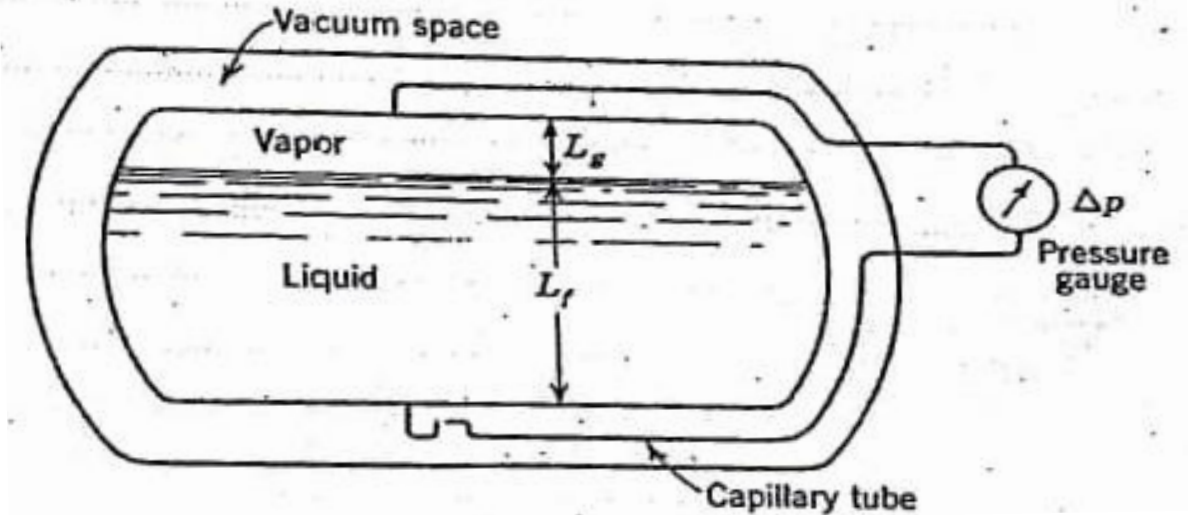
$$\Delta p = \rho_f L_f g / g_c + \rho_g L_g g / g_c$$

L_f = height of the liquid column
$L_g = L - L_f$ = height of the vapor column
L = inside diameter of the vessel
ρ_f = liquid density
ρ_g = vapor density (saturated)
g = local acceleration due to gravity
g_c = conversion factor in Newton's Second Law of Motion

$$\Delta p = (\rho_f - \rho_g) L_f g / g_c + \rho_g L g / g_c$$

$$\Delta p = \rho_f L_f g / g_c$$

$$S_0 = \frac{d(\Delta p)}{dL_f} = (\rho_f - \rho_g)\frac{g}{g_c}$$

Electric Resistance Gauges

- Heat transfer coefficient of liquid is 1 or 2 orders of magnitude higher than that of vapor.

- For the same heat transfer rate, wire will have greater T in vapor.

- Resistance varies with temperature which can be measured.

- For vertical arrangement resistance may be directly calibrated against level. But energy dissipated is more.
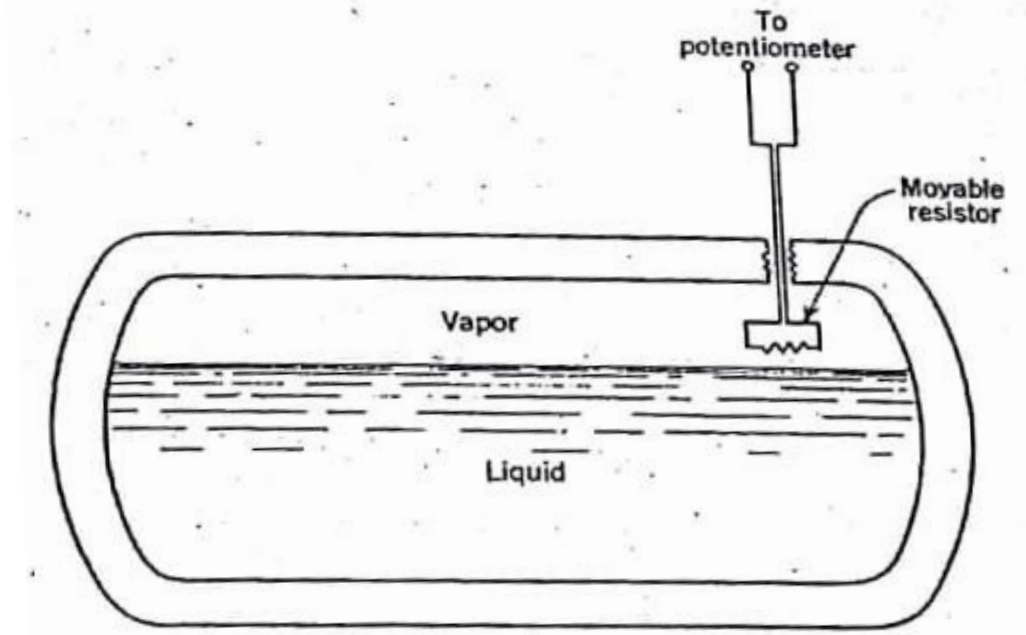
$$h_c = (2k_l/D)/\ln(1 + \phi N_{Gr}^{-1/4})$$

where $\phi = 3.70(N_{Pr} + 0.952)^{1/4} N_{Pr}^{-1/2}$

$N_{Gr} = g\beta_l \rho^2 D^3 \Delta T/\mu^2 = $ Grashof number

$N_{Pr} = $ Prandtl number
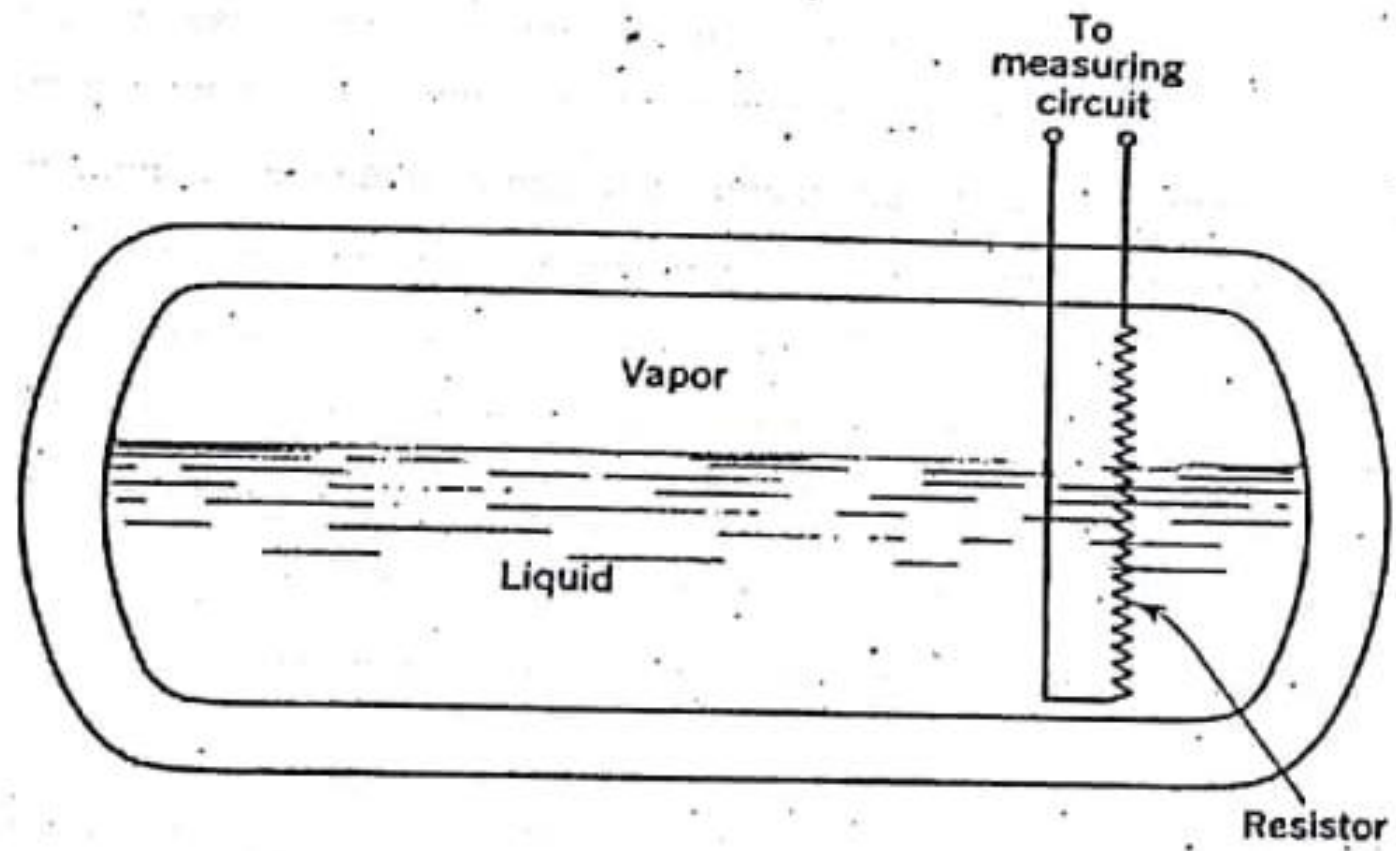
$k_l = $ thermal conductivity

$D = $ wire diameter

$\beta_l = $ thermal expansion coefficient

$\rho = $ density

$\mu = $ viscosity

$\Delta T = $ temperature difference between wire and fluid

To measuring circuit

Vapor

Liquid

Resistor

Capacitance Probes

- Same way as capacitance quality probe.
- Concentric cylinder capacitor placed vertically in the container.
- Total capacitance is found by the given equation.
- Can be solved for liquid level (L_f). Liquid level can be measured from the Capacitance value.
- Or level can be calibrated directly against capacitance.
- Sensitivity is independent of liquid level and length of the probe.
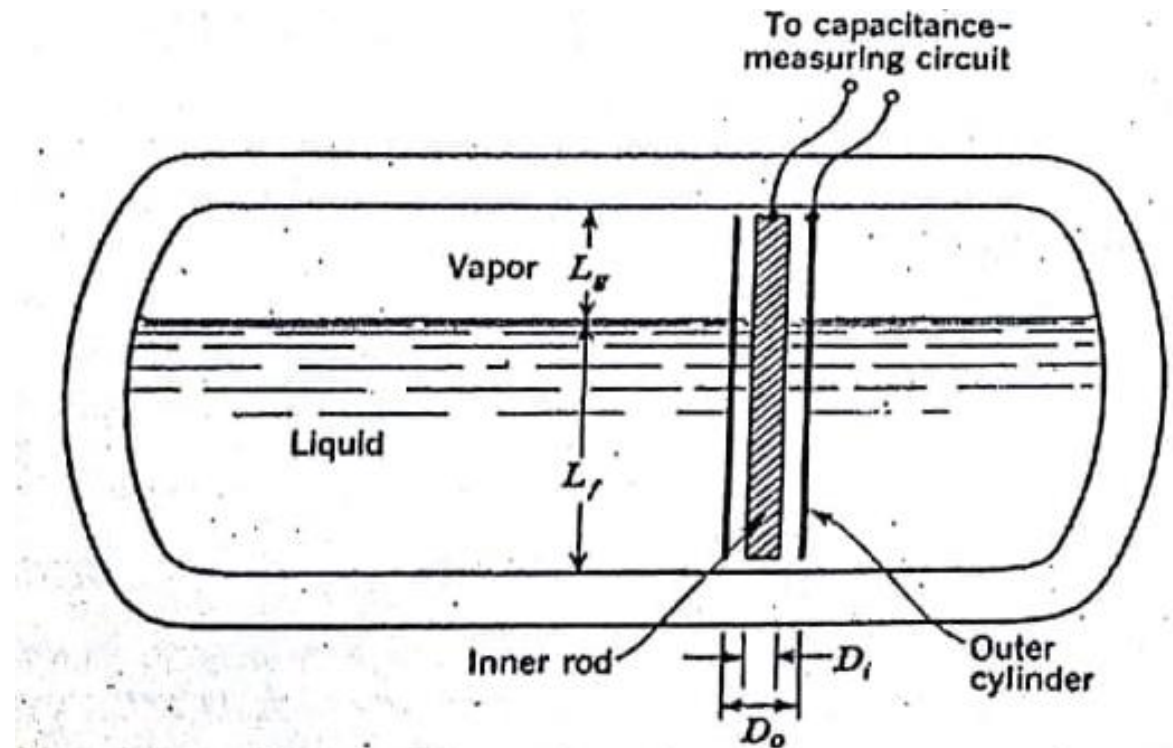
$$C = C_f + C_g = \frac{2\pi(L_f \epsilon_f + L_g \epsilon_g)\, \epsilon_0}{\ln(D_o/D_i)}$$

$$C = \frac{2\pi L \epsilon_0}{\ln(D_o/D_i)}\left[\epsilon_g + (\epsilon_f - \epsilon_g)\left(\frac{L_f}{L}\right)\right]$$

$$L_f = \frac{C \ln(D_o/D_i)}{2\pi(\epsilon_f - \epsilon_g)\epsilon_0} - \frac{\epsilon_g L}{\epsilon_f - \epsilon_g}$$

$$S_0 = \frac{dC}{dL_f} = \frac{2\pi(\epsilon_f - \epsilon_g)\epsilon_0}{\ln(D_o/D_i)}$$

Thermodynamic Gauge

- Based on the fact that liquid undergoes large change in volume when evaporated.
- Thin heated capillary tube (inserted in the container) connected to a dead volume (at ambient condition) and a pressure gauge.
- Initially charged with the same gas as in storage.
- Condenses when immersed reducing dead volume pressure.
- Condensation is proportional to the liquid level.
- Hence liquid level can be calibrated against pressure. Or by the following relation. (A_c – area of capillary)
- Density can be found from the modified ideal gas relation using supercompressibility factor (z)
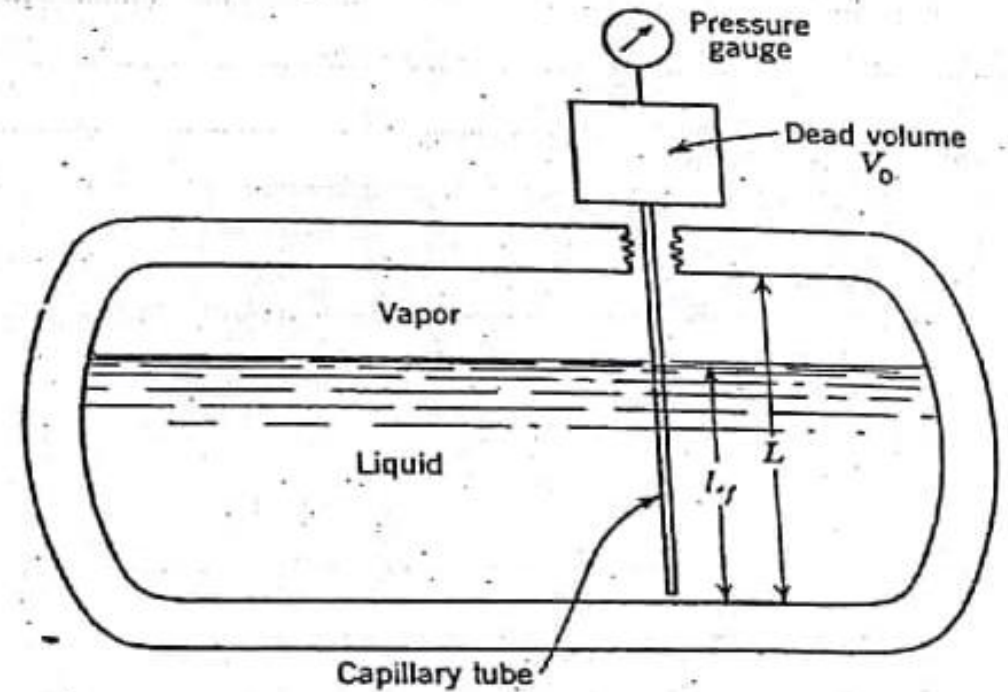
$$m = m_f + m_g + m_0 = \rho_f V_f + \rho_g V_g + \rho_0 V_0$$

$$m = \rho_f A_c L_f + \rho_g A_c L_g + \rho_0 V_0$$

$$m = A_c[(\rho_f - \rho_g)L_f + \rho_g L] + \rho_0 V_0$$

$$\frac{L_f}{L} = \frac{[(m - \rho_0 V_0)/A_c L] - \rho_g}{\rho_f - \rho_g}$$

$$p = Z \rho R T$$